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ABSTRACT

The second Indiana Mathematics Contest included tests in Algebra (First Course), Geometry, Algebra (Second Course), and Comprehensive. Test-writing responsibilities were delegated to mathematics and mathematics education faculties at four state universities, with the mathematical content based on course objectives in the state guidelines. Fourteen sites across Indiana served as test centers, with 2394 students participating. Outstanding Scholar awards were presented to 136 students. Analysis of the performance of a random stratified sample of 129 students on selected items from each test is then presented. Item difficulties were examined to determine areas of weakness. For Algebra (First Course), fundamental operations on rational expressions and roots for quadratic equations were of particular concern. Three areas of concern were evident on the Algebra (Second Course) results: exponential and logarithmic functions, permutations and combinations, and ability to analyze conditions related to solutions to elementary algebraic equations and functions. In Geometry, ability to apply information in novel situations seemed to create difficulty, rather than specific content. On the Comprehensive Test, concern was with a range of content. Overall, a lack of problem-solving ability was noted. Appendices contain the four tests. (MNS)

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THE 1984 INDIANA STATE MATHEMATICS CONTEST: TESTS AND ANALYSIS OF STUDENT PERFORMANCE

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INTRODUCTION

The second Indiana State High School Mathematics Contest was held in April of 1984. Sponsored by the Indiana Council of Teachers of Mathematics with funding from the Lilly Endowment, Inc., the contest is a revival of a state-sponsored event held annually until the early 1960s.

Preliminary planning for the 1983 contest started in 1981 through a group of interested members of the Indiana Council. This first contest featured an Algebra (First Course) test and a Comprehensive test, with major goals being:

1) To stimulate interest in the study of mathematics;
2) To recognize outstanding mathematics students;
3) To foster communication among mathematics students;
4) To facilitate communication within the mathematics education community in Indiana;
5) To promote appreciation of mathematical excellence;
6) To recognize outstanding mathematical achievement.

With over 1700 students participating, the contest was deemed a success and plans were established for a 1984 contest to include tests in Algebra (First Course), Geometry, Algebra (Second Course), and Comprehensive. Only one major change was made in the rules. To help alleviate the large number of students who registered for the contest but did not participate, a $1 registration fee was implemented. The Contest Committee remained intact, with Ball State University, Muncie, serving as the coordinating agency.

TEST CONSTRUCTION

Test writing responsibilities were delegated to mathematics and mathematics education faculties at four state universities: Purdue University - North Central, Algebra (First Course); Indiana University - Bloomington, Geometry; Ball State University, Algebra (Second Course); and Indiana University - Purdue University - Fort Wayne, Comprehensive. The mathematical content of each test was based on course objectives listed in Guidelines for Mathematics Instruction in Indiana Schools (1977), published by the Indiana Department of Public Instruction. Items were solicited from classroom teachers throughout the state or were formulated by the test writers. Using standard test construction procedures, the four

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multiple-choice (90 minute time limit) tests were finalized with the following number of items:

Algebra (First Course)	75 items
Geometry	30 items
Algebra (Second Course)	50 items
Comprehensive	28 items

TEST ADMINISTRATION

Fourteen sites across the state served as test centers, with a total of 2394 students participating. Each site coordinator was responsible for administering and grading tests, presenting certificates to all participants and "Scholar" awards to those students who correctly answered 75 per cent of the test questions, and arranging any local program for students.

All test results were sent to Ball State University for analysis and to determine students scoring in the top five per cent on each test. These students received a special certificate designating them as "Outstanding Scholars." Award cut-off scores for Algebra (First Course), Geometry, Algebra (Second Course), and Comprehensive were, respectively, 55, 24, 32, and 21. Table 1 below provides information on student participation at each site for each test.

Table 1
1984 Indiana State Mathematics Contest
Student Participation

Site	Test			
	Algebra (1st)	Geometry	Algebra (2nd)	Comprehensive
Ball State	55	29	22	32
Butler	117	81	75	62
Franklin	42	64	37	32
Indiana State	99	99	48	26
I.S.U. Evansville	83	47	32	37
I.U. Bloomington	29	11	13	10
I.U. Richmond	55	18	25	10
I.U. Kokomo	67	36	43	43
I.U. Gary	84	64	52	35
I.U.P.U. Ft. Wayne	53	34	25	25
I.U. New Albany	31	46	24	53
Purdue	35	43	53	39
Purdue Westville	86	33	14	18
Saint Mary's	24	30	12	26
Total	860	635	465	448

The remainder of this article presents an analysis of student performance on selected items of each test, based on a 15 per cent proportional stratified random sample. Sample sizes for each test at each site are presented in Table 2. Complete copies of all four tests are included in the Appendix.

Table 2
Sample Sizes by Test Site and Test

Site	Test			
	Algebra (1st)	Geometry	Algebra (2nd)	Comprehensive
Ball State	8	4	3	5
Butler	18	12	11	9
Franklin	6	10	6	5
Indiana State	15	15	7	4
I.S.U. Evansville	12	7	5	6
I.U. Bloomington	4	2	2	2
I.U. Richmond	8	3	4	2
I.U. Kokomo	10	5	6	6
I.U. Gary	13	10	8	5
I.U.P.U. Ft. Wayne	8	5	4	4
I.U. New Albany	5	7	4	8
Purdue	5	6	8	6
Purdue Westville	13	5	2	3
Saint Mary's	4	4	5	4
Total	129	96	72	69

The analysis focused on the following questions:

1. What do students know?
2. What don't students know?
3. What items make a difference in student performance?

Operationally, answers to each question were found, respectively, by:

1. An item difficulty of .70 or higher.
2. An item difficulty of .25 or less.
3. A discrimination index of .59 or higher.

Algebra (First Course)

On the 75 item Algebra (First Course) test, the range of scores for all students participating (N = 860) was from 7 to 71, with a sample mean of 36.57 and a standard deviation of 11.52 (n = 129). Outstanding Scholar awards were presented to 48 students.

In the analysis, 11 questions produced item difficulties of .70 or greater, providing an indication of what students do know. According to Bloom's Taxonomy of Cognitive Objectives, these items fell into three levels: Knowledge (Items 3, 43, 47); Comprehension (Items 1, 10, 12, 18, 27); and Application (41, 44, 49). Identification was the major focus in the knowledge category: "Identify" a rational number, a binomial, a prime number. Simple procedures (union and intersection, adding and multiplying algebraic expressions, rules for exponents) were tested by the items in the Comprehension category. The three Application items involved arithmetic averages, solving a system of two equations, and specifying the direction of a line (positive or negative slope) containing two points.

To determine what algebra students do not know, questions with item difficulties of .25 or less were examined. Ten items were in this category, four at the Comprehension level (Items 14, 48, 51, 59) and six at the Application level (Items 8, 11, 26, 57, 66, 75).

Two content areas emerge as areas for concern:

1) Fundamental operations on rational expressions, and
2) Roots and the nature of roots for quadratic equations.

Students were generally unsuccessful in finding least common denominators for equations with rational expressions, in order to solve for a specific variable. They also had difficulty in determining a quadratic equation with given irrational roots, and in determining the sum of the roots for a quadratic equation.

There were 10 items which discriminated the highest scoring students from the lowest scoring students. Again, four items were at the Comprehension level (Items 6, 20, 25, 50) and six were at the Application level (Items 21, 22, 36, 37, 54, 61). One identifiable area for discrimination was solving equations in one variable, including quadratic equations and proportions. Two items dealing with exponents also were discriminators.

In summarizing the overall results of the test, it is apparent that students experience difficulty with any problem involving rational expressions. They can find a least common denominator for two rational expressions (Item 30), but cannot apply this procedure in more complex problem situations with equations. Students do have a good grasp of simple algebraic procedures, such as combining like terms, multiplying polynomials, and applying rules for exponents. However, several concepts or procedures that students do know according to test results were notions that often had been previously learned in junior high school or intermediate grade mathematics (prime number, ordering rational numbers, averaging, union and intersection).

Algebra (Second Course)

On the 50-item Algebra (Second Course) test, the range of scores for participating students (N = 475) was from 7 to 44 with a sample mean of 21.25 and a standard deviation of 7.35 (n = 72). Outstanding Scholar awards were presented to 27 students.

An item analysis identified four questions having item difficulties of .70 or greater (Items 16, 18, 38, 43). These test items indicate that students generally possess adequate knowledge for successful work in elementary sequences, variation, and simple systems of linear equations. Using Bloom's Taxonomy of Cognitive Objectives, these four test items were found to be at the Application level.

Questions with item difficulties less than .25 were examined to determine what the students did not know. There were 14 test items in this category; one at the Knowledge level (Item 12), four at the Comprehension level (Items 24, 26, 49, 50), four at the Applications level (Items 8, 21, 22, 44), four at the Analysis level (Items 7, 34, 35, 39), and one at the Synthesis level (Item 19). These items identify three content areas of concern:

1. basic understanding of exponential and logarithmic functions.
2. fundamental knowledge of permutations and combinations, and
3. ability to analyze conditions relating to solutions to elementary algebraic equations or to algebraic functions.

Students were generally unsuccessful in solving logarithmic and exponential equations. They also had considerable difficulty in determining the nature of solutions to a system of equations as well as determination of a coefficient to obtain a specified solution to a polynomial equation. Additionally, students generally were unsuccessful in solving permutation/combination problems.

There were 14 test items which discriminated between the highest scoring and lowest scoring students. Applying the Bloom Taxonomy to these, there were three items at the Comprehension level (Items 17, 25, 42), five at the Applications level (Items 1, 4, 5, 13, 20), five at the Analysis level (Items 33, 37, 40, 41, 48), and one at the Synthesis level (Item 32). The one area clearly identified by the discrimination index was that of algebra of polynomials, including solution of polynomial equations, zeros of polynomial functions, and factorization of polynomial expressions.

In summary, it is apparent that students have difficulty with the area of polynomial algebra, especially if an analysis of the problem is required in order to obtain a solution. Only two-thirds of the students could correctly solve a non-standard quadratic equation, and only 15 percent of the students could do so if the quadratic equation was one of exponential form. The undesirable

results in the area of logarithmic and exponential functions, permutations, and combinations may be due to the fact that these had not yet been taught (even though the contest was held quite late in the school year). Nevertheless, the results indicate that the types of problems considered in the second algebra course should be broadened in scope to include some examples of what are currently considered non-standard types.

GEOMETRY

The Geometry Test consisting of 30 items was taken by 635 students, with scores ranging from 4 to 28, a sample mean of 14.10 and a standard deviation of 4.89 (n = 96). Outstanding Scholar awards were presented to 40 students.

Six items had difficulty indices greater than .70. While the content varies among these items, they all deal with content that seems quite basic. An analysis of item level following Bloom's Taxonomy indicates that these items are mostly at the comprehension level, with some at the knowledge level.

Four items had difficulty indices below .20. Item 19 requires only recognition of properties of similar right triangles with shared angles. A closer look at the item statistics shows that the answer key was wrong and that actually 74 percent of the sample chose the correct answer. We may assume then that this content in this context was in fact mastered. (This error in test construction, involving one item, could not reasonably have effected the identification of Outstanding Scholars.)

Items 2, 10, and 24, however, were simply missed by the bulk of the students. While Items 2 and 10 both involve polygonal properties, the first requires students to coordinate the relationships between the sum of the angles of a triangle and the sum of the external angles of a polygon, and Item 10 concerns the area and perimeter relationships among a regular hexagon and its component regular triangles. In even greater contrast, Item 24 involves the areas of circular sections. It seems unlikely then that the basic content provided the difficulty. Rather, the difficulty comes with the relationships among these content pieces. The problem, in a phrase, was problem solving.

The five best discriminators were Items 6, 8, 12, 21, and 23. All five items deal with essentially different content. Two of them are classified as Problem Solving items, two as Comprehension / Application items, and one, dealing with the volume of a cone, as Knowledge.

It is difficult to sort through these items and statistics to decide what content geometry Indiana students know best or least. One would expect a typically good (not necessarily great) geometry class to be able to solve all the problems on this test. While the test provides some evidence that specific topics might be learned by some students rather than others (e.g., volume of a cone), the

implications of these data for teachers have more to do with the approach to content than with content itself. The difficult items and those that tended to make the difference are, indeed, those problems that require a student to not only know a piece of information, but to recognize it in novel situations. Good problems require this.

COMPREHENSIVE

The Comprehensive Test consisting of 28 items was taken by 448 students, with scores ranging from 1 to 27, a sample mean of 11.64 and a standard deviation of 4.63 (n = 69). Outstanding Scholar awards were presented to 21 students.

In analyzing what students know, only Items 4 and 10 were correctly answered by 70 percent or more of the sample. Closely after these two items were Item 3 (.67), Item 25 (.59), and Item 23 (.58). The three easiest items were concerned with geometry (Pythagorean Theorem/Special Triangles), with prime factorization, and with the solution of simple second degree polynomial equaton. The remaining two dealt with special triangles and the solution of a linear equation based upon arithmetic progression. These last two items were also among the best discriminators of the test.

Items 11, 20, and 27 had difficulty indices less than chance (.20), while Items 5 and 21 had difficulty indices less than .25.

Item 27 was not answered correctly by any student in the sample. An analysis of the student responses indicates that students identified an easy special case, or partial answer, as the correct answer. The problem is made complex by the use of the logical descriptors "all" and "some." (Note that -1 raised to any even power is 1.) It may be that this item is a trick question resulting from mixing an obvious answer that is wrong, with a subtle but correct answer, or it may identify an area of concern.

Item 11 also failed to provide any discrimination, and so identifies another area of concern: understanding the implications of the prime factorization theorem.

Item 20, with a respectable discrimination index, involves both simultaneous solutions and the recognition of factors of the sum of two cubes. An analysis of the wrong choices indicates the misuse of coefficient laws, or an error in sign.

Items with discrimination indices greater than .59 were Items 6, 8, 22, and 26 (all at .67). With indices of .56, Items 16, 23, and 25 should also be considered in the set of items that made a difference.

In summary, the Comprehensive Test appears to be a reasonably reliable measure of student achievement. The items that made a difference represented a wide range of content. There is some bias in favor of those students who have had the opportunity to study analytic geometry and trigonometry.

GENERAL SUMMARY

The results of these tests provide insight into what our students have learned and insight into problem solving. Overall, there was a lack of problem solving ability on the part of the participants for each test. The analysis shows students experienced difficulty in several instances on items requiring more than one step to reach a solution. It is hoped that the contest will have the effect of not only encouraging teachers to investigate a broad range of content, but to do so in contexts in which the reasons for and the structure of the knowledge are made evident. For special assignments, for in-classs activities, and for mathematics club meetings, the tests are excellent sources to help promote problem solving across the high school mathematics curriculum.

APPENDIX

1984

STATE HIGH SCHOOL MATHEMATICS CONTEST

SPONSORED BY THE INDIANA COUNCIL OF TEACHERS OF MATHEMATICS

PARTIALLY FUNDED BY THE LILLY ENDOWMENT, INC.

ALGEBRA TEST

(FIRST COURSE)

THIS TEST WAS PREPARED BY THE MATHEMATICS EDUCATION FACULTY AT PURDUE UNIVERSITY - NORTH CENTRAL.

DIRECTIONS FOR TEST:

DO NOT open this booklet until you are told to do so.

This is a test of your competence in high school algebra, first course. For each of the 75 problems there are listed 5 possible answers. You are to work each problem and determine which is the correct answer, and indicate your choice by filling in the circle in the correct place on the separate answer sheet provided. A sample follows:

1. If $x + 2 = 6$, then x equals:

   A. 8

   B. 3

   C. $\frac{1}{3}$

   D. 4

   E. none of these

1. A B C D E
   ① ② ③ ● ⑤

The correct answer for the sample is "4", which is answer D; therefore, you should answer this question by filling in the circle D as indicated above.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any question. If you are unable to work any particular problem, it is to your advantage to guess at the answer rather than leave it blank. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet and begin work. When you have finished one page, go on to the next page. The working time for the entire test is 90 minutes.

DIRECTIONS FOR ANSWER SHEET:

Fill in your name in the blanks provided. Above your name write the name of your school and the city where it is located, including zip code.

Along the side of your name indicate your sex and grade by filling in the circle provided. A sample follows:

SAMPLE: Mary A. Brown who goes to Western High School in Muncie and is in the ninth grade would write across the top and fill in along the side.

WESTERN HIGH SCHOOL - MUNCIE, INDIANA 47306

NAME (Last, First, M.I.)														
B	R	O	W	N		M	A	R	Y		A			

SEX (M) ●

GRADE ● ⑩ ⑪ ⑫

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.

STATE MATH CONTEST - ALGEBRA I EXAM - PAGE 1

1. Simplify: $\frac{a^5b^0c^2}{a^2b^3c^7}$

a) $\frac{a^3b^2}{c^5}$ b) $\frac{a^3}{b^3c^5}$ c) $a^7b^3c^9$ d) $\frac{a^3}{b^2c^5}$ e) None of these

2. Solve for b: $A = \frac{1}{2}bh$

a) $\frac{2A}{h}$ b) $\frac{A}{2h}$ c) $\frac{2h}{A}$ d) $2A - h$ e) None of these

3. Which of the following is a rational number?

a) $\sqrt{12}$ b) $\sqrt{20}$ c) $\sqrt{81}$ d) $\sqrt{\frac{1}{2}}$ e) None of these

4. The sum of the digits of a two-digit number is 10. If the digits are reversed, the new number exceeds the old by 36. Find the number.

a) 37 b) 46 c) 73 d) 64 e) None of these

5. Factor completely: $x^4 - y^4$

a) $(x^2 + y^2)(x^2 - y^2)$ b) $(x + y)^2(x - y)^2$

c) $(x^2 - y^2)^2$ d) $(x^2 + y^2)(x + y)(x - y)$

e) None of these

6. What is the y-intercept for the line represented by the equation $3x - 5y = 15$?

a) $(0,-3)$ b) $(0,5)$ c) $(0,\frac{-3}{5})$ d) $(0,15)$ e) None of these

7. Simplify Completely: $\sqrt{32} + \sqrt{50} + \sqrt{72}$

a) $\sqrt{154}$ b) $5\sqrt{8} + 5\sqrt{2}$ c) $15\sqrt{2}$

d) $2\sqrt{8} + 5\sqrt{2} + 2\sqrt{18}$ e) None of these

8. An equation of line k is $2x - y = 5$. An equation of a line perpendicular to k is

a) $2x - y = 7$ b) $2x + 4y = 13$ c) $y - 2x = 5$
d) $2x + y = 7$ e) None of these

9. Simplify completely:
$(3x^2 + 5x - 7) - (2x^3 + 5x^2 + 7x + 6) + (x - 2x^2 + 7)$

a) $2x^3 + 4x^2 + 13x - 20$ b) $-4x^2 - x - 6$
c) $-2x^3 + 6x^2 - x + 6$ d) $-2x^3 - 4x^2 - x + 6$
e) None of these

10. Given: $A = \{2,4,6,8\}$, $B = \{1,3,5,7\}$, $C = \{4,5,10\}$.
Find: $(A \cup B) \cap C$

a) $\{4,5\}$ b) $\{4,5,6\}$ c) ϕ d) $\{4,5,10\}$ e) None of these

11. The equation whose roots are $2 + \sqrt{3}$ and $2 - \sqrt{3}$ is

a) $x^2 - 4x + 1 = 0$ b) $x^2 - 4x - 5 = 0$
c) $x^2 + 4x - 5 = 0$ d) $x^2 + 4x - 1 = 0$ e) None of these

12. Multiply and simplify completely: $(3x - 1)(x^2 + 2x - 4)$

a) $3x^3 + 7x^2 - 14x + 4$ b) $3x^3 + 5x^2 - 14x + 4$
c) $3x^3 + 4x^2 - 10x + 4$ d) $3x^3 + 6x^2 - 10x + 4$
e) None of these

13. A collection of 28 coins, some nickels and some dimes, totals $2.20. How many dimes are there?

a) 14 b) 12 c) 16 d) 10 e) None of these

14. The sum of the roots of the equation $x^2 - 3x - 5 = 0$ is:

a) -5 b) -3 c) 3 d) 5 e) None of these

15. Find the solution set for x where $x^2 - 4x + 7 = 3x - 5$

a) {3,4} b) {-3,-4} c) {3,-4} d) {-3,4} e) None of these

16. $\frac{6m^2}{n} \cdot \frac{2n}{3m} =$

a) 2m b) 4m c) $\frac{4m}{n}$ d) 4mn e) None of these

17. If $\frac{9}{n} - \frac{7}{n} = \frac{1}{6}$, then n =

a) $\frac{1}{12}$ b) $\frac{1}{3}$ c) 3 d) 12 e) None of these

18. If R = {-3,-2,-1} and S = {-3,-1,1,3}, find R ∪ S.

a) {-3,-2,-1,1,3} b) {-3,-1} c) ϕ
d) {-3,3} e) None of these

19. Solve for x: $7 - 3x \leq 15 + x$

a) $x \leq -2$ b) $x \geq -2$ c) $x \leq 2$ d) $x \geq 2$ e) None of these

20. $2c(2c^3)^2 =$

a) $4c^7$ b) $8c^6$ c) $8c^7$ d) $4c^6$ e) None of these

21. For what values of x is $\frac{x}{6} = \frac{1}{2}(x - 3) - \frac{x}{3}$ true?

a) 0 only b) 3 only c) 0 and 3 only
d) all values e) None of these

22. If $\frac{x - 5}{8x} = \frac{3}{x + 5}$, then the solution set for x is

a) {1,-5,5} b) {0,-5,5} c) {-5,5} d) {25,-1} e) None of these

23. Simplify $\sqrt{45x} \cdot \sqrt{3x^3}$

a) $135x^4$ b) $135x^2$ c) $x^2\sqrt{135}$ d) $3x^2\sqrt{15}$ e) None of these

24. For what values of k will the roots of the equation $x^2 + 6x + k = 0$ be equal?

a) 3 b) 6 c) 9 d) 36 e) None of these

25. The expression $|x - 1| = 1$ implies that

a) x is between 0 and 2 b) x is either 0 or 2
c) x is less than 2 d) x is 0
e) None of these

26. If $\frac{1}{x^3} - \frac{1}{x^2} - \frac{1}{x} - 1 = 0$, find the value of $x^3 + x^2 + x + 1$.

a) -2 b) -1 c) 2 d) 0 e) None of these

27. $(3a^3 - 4a^2b + 7b^2) + (2a^3 + 8ab^2 - 2b^2) =$

a) $5a^3 + 4a^2b^2 + 5b^2$ b) $5a^3 - 4a^2b + 8ab^2 + 5b^2$

c) $5a^6 + 4a^3b^3 + 5b^4$ d) $5a^3 - 4a^2b + 8ab^2 + 9b^2$

e) None of these

28. The degree of the polynomial $2a^2b^3 - 3a^3b + a^2b^2$ is

a) 2 b) 3 c) 4 d) 5 e) None of these

29. Find two positive integers such that the difference of their squares is 40 and the square of the greater is 4 more than 5 times the square of the lesser

a) 9 and 49 b) 11 and 51 c) 3 and 7

d) 9 and 13 e) None of these

30. The least common denominator of the fractions $\frac{1}{s^2 + s - 12}$ and $\frac{1}{s^2 - 5s + 6}$ is

a) $(s - 2)(s - 3)(s + 4)$ b) $(s - 2)(s - 3)^2(s + 4)$

c) $(s - 2)(s + 3)(s - 4)$ d) $(s - 2)(s - 3)(s + 3)(s - 4)$

e) None of these

31. If $x = \frac{A}{1 + A}$ and $y = \frac{1 - A}{A}$, the value of $\frac{x - y}{x + y}$ in terms of A is

a) $A - 1$ b) -1 c) 1 d) $2A^2 - 1$ e) None of these

32. If Sam can roof a house alone in 6 days, Bill can roof the same house alone in 4 days, and Pete can roof it by himself in 8 days, how many days will it take them to roof the house working together?

a) 2 days b) $1\frac{11}{13}$ days c) $1\frac{5}{9}$ days d) $1\frac{2}{11}$ days e) None of these

33. If A and B are real numbers and $AB > 0$, which is never true?

a) $A > B$ b) $A > 0$ and $B < 0$ c) $A > 0$ and $B > 0$

d) $A < 0$ and $B < 0$ e) None of these

34. The roots of $3x^2 - 4x = 6$ are

a) $\frac{-4 \pm \sqrt{88}}{2}$ b) $\frac{4 \pm \sqrt{22}}{2}$ c) $\frac{2 \pm \sqrt{22}}{3}$

d) $\frac{2 \pm \sqrt{88}}{3}$ e) None of these

35. In which quadrant do the graphs of the equations $3x - y = -15$ and $y = x + 1$ intersect?

a) I b) II c) III d) IV e) None of these

36. 14.2 is 20% of what number?

a) 71 b) 2.84 c) 28.4 d) .00140845 e) None of these

37. Which polynomial cannot be factored?

a) $a^2 - b^2$ b) $a^2 + 2ab + b^2$ c) $a^2 + b^2$

d) $a^2 - 2ab + b^2$ e) None of these

38. $a^5 \cdot a^4$ (where $a \neq 0$) =

a) a^{54} b) a^9 c) a^{20} d) a^{-1} e) None of these

39. The sum of $3\sqrt{8}$ and $2\sqrt{2}$ is

a) 24 b) $14\sqrt{2}$ c) $12\sqrt{2}$ d) $8\sqrt{2}$ e) None of these

40. The excluded value(s) of the variable in the fraction $\frac{x - 3}{x^2 + 4x}$ is (are)

a) 3 b) -2,2 c) 0,-4 d) -4,0,3 e) None of these

41. Solve this system for x and y: $3x - y = 13$
$x + 2y = 9$

a) $x = -5, y = 2$ b) $x = 5, y = -2$ c) $x = 5, y = 2$

d) $x = -5, y = -2$ e) None of these

42. Simplify: $(-3xy^2z^6)^5$

a) $-15x^5y^7z^{11}$ b) $15x^5y^{10}z^{30}$ c) $-243x^5y^7z^{11}$

d) $-243x^5y^{10}z^{30}$ e) None of these

43. Classify the polynomial $3x - 7$.

a) monomial b) binomial c) trinomial
d) quadratic e) None of these

44. The average of a set of numbers is 6. If the sum of the set of numbers is 144, how many numbers are in the set?

a) 24 b) 18 c) 12 d) 6 e) None of these

45. If p varies directly as s and $p = 8$ when $s = 4$, what is the value of p when $s = \frac{1}{2}$?

a) 8 b) 4 c) 2 d) 1 e) None of these

46. Solve: $4 < 2(3x + 4) < 40$

a) $\frac{2}{3} < x < \frac{-16}{3}$ b) $\frac{-2}{3} < x < \frac{16}{3}$ c) $\frac{2}{3} > x > \frac{-16}{3}$

d) $\frac{-2}{3} > x > \frac{16}{3}$ e) None of these

47. Which number is prime?

a) 21 b) 31 c) 51 d) 81 e) None of these

48. How many real numbers are reciprocals (multiplicative inverses) of themselves?

a) none b) 1 c) 2 d) 3 e) None of these

49. The slope of the line through the points (3,4) and (6,8) is

a) positive b) negative c) 0
d) undefined e) None of these

50. $(-1)^{117}$ =

a) -117 b) 117 c) 1 d) -1 e) None of these

51. If $0 < x < 1$, then

a) $x^2 > x$ b) $2x < x$ c) $\frac{1}{x} > 1$ d) x is negative e) None of these

52. The equation of a horizontal line, four units above the x-axis is

a) $x + y = 4$ b) $y = 4$ c) $x - 4 = 0$
d) $y > 4$ e) None of these

53. The statement $a^2 + b^2 = 0$ is equivalent to

a) Neither a nor b is zero b) $a = 0$ and $b = 0$
c) a and b have opposite signs d) $a \cdot b = 0$
e) None of these

54. The equation $2(x + 1) = 2x + 4$ has how many solutions?

a) 1 b) 2 c) None d) More than 2 e) None of these

55. Which statement is correct?

a) $\frac{6}{7} > \frac{31}{35}$ b) $\frac{1}{6} > \frac{1}{5}$ c) $-8 > -7$ d) $\sqrt{15} > 4$ e) None of these

56. The values of x and y, if $y - x = 13$ and $3x + 2y = 36$, are

a) (-2,15) b) (2,-15) c) (2,15) d) (15,2) e) None of these

57. If $C \neq 0$ and $G \neq 0$, then solve for C: $\frac{V^2}{2G} + \frac{P}{C} = H$

a) $C = \frac{2G(CH - P)}{V^2}$ b) $C = \frac{2PG}{2GH - V^2}$ c) $C = PH - \frac{V^2}{2G}$

d) $C = P(H - \frac{2G}{V^2})$ e) None of these

58. Which of the following pairs (x,y) is not in the intersection of the graphs $x - y \geq 2$ and $x + y \leq 3$?

a) (2,-1) b) $(\frac{5}{2},\frac{1}{2})$ c) $(\pi,-1)$ d) $(\sqrt{2},\sqrt{2})$ e) None of these

59. The expression $-2 + \{4 - 3(-2 - 4)\} \div \{2 + 3 - 2(1 + 4)\} =$

a) -4 b) $\frac{20}{5}$ c) $6\frac{1}{2}$ d) $\frac{-32}{5}$ e) None of these

60. The expression $(x - 3)(2 - x) - (3 + x)(2 + 3x)$, when simplified, is equal to

a) $-4x^2 - 6x - 12$ b) $-2x^2 + 16x$ c) $2x^2 + 16x - 12$

d) $-4x^2 - 5x$ e) None of these

61. One root of the equation $x^2 + 6x - 16 = 0$ is

a) 16 b) -2 c) 8 d) -8 e) None of these

62. The average of the numbers represented by $N + 3$, $2N - 1$ and $3N + 4$ is

a) $\frac{5N + 6}{3}$ b) $2N + 2$ c) $3N + 3$ d) $\frac{6N + 7}{3}$ e) None of these

63. The tens digit of a two digit number is twice the units digit. If the digits of the number are reversed, the original number is 12 less than twice the result. What is the orignal number?

a) 96 b) 84 c) 63 d) 42 e) None of these

64. If $x > y$, then which statement is always true?

a) $x^2 > y^2$ b) $y^2 > x^2$ c) $x > 0$ d) $y - x < 0$ e) None of these

65. If $x \neq 0$ then $x =$

a) $\frac{x}{0}$ b) $\frac{0}{x}$ c) $\frac{1}{x}$ d) $x + 0$ e) None of these

66. The distance between the points $(-2,3)$ and $(2,6)$ is

a) 9 b) 25 c) 7 d) 5 e) None of these

67. The quadratic equation $4x^2 - 12x + 9 = 0$ has how many <u>real</u> solutions (roots)?

a) 1 b) 2 c) None d) More than 2
e) None of these

68. Which set is <u>not</u> closed under multiplication?

a) The even integers: {...,-4,-2,0,2,4,...} b) {0,-1,1}

c) Positive real numbers

d) The prime integers e) None of these

69. $(-3)^2 + 20 \div 4 + 1 =$

a) 15 b) -3 c) 13 d) -5 e) None of these

70. Which expression concerning radicals is true?

a) $\sqrt{a} + \sqrt{b} = \sqrt{a + b}$ b) $\sqrt{a - b} = \sqrt{a} - \sqrt{b}$ c) $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$

d) $\sqrt[3]{\sqrt{x}} = \sqrt[5]{x}$ e) None of these

71. If $\frac{5}{8}$ of a gallon of milk costs \$1.15, then $\frac{3}{4}$ of a gallon will cost

a) \$1.18 b) \$1.58 c) \$1.28 d) \$1.38 e) None of these

72. If $b*c = \frac{2b - c}{b - 2c}$, then find in simplest form $6*2$.

a) 5 b) 3 c) 8 d) 2 e) None of these

73. If a plane's air speed is s km/h and there is a 50 km/h head wind, how many hours will it take to fly 650 km?

a) $\frac{650}{s - 50}$ b) 13 c) $\frac{700}{s}$ d) $\frac{650}{s + 50}$ e) None of these

74. Which of the following is a solution of the system

$$\begin{aligned} 2x - 2y + z &= -4 \\ x + y - 3z &= -9 \\ 2y - x - z &= 1 \end{aligned}$$

a) (3,0,2) b) (3,1,-8) c) (-8,-7,-2) d) (-3,0,2) e) None of these

75. The solution set of $\frac{30}{T^2 - 9} + 2 = \frac{5}{T - 3}$ is

a) $\{-\frac{1}{2}, 3\}$ b) $\{-\frac{1}{2}\}$ c) $\{3\}$ d) $\{-3, \frac{1}{2}\}$ e) None of these

1984

STATE HIGH SCHOOL MATHEMATICS CONTEST

SPONSORED BY THE INDIANA COUNCIL OF TEACHERS OF MATHEMATICS

PARTIALLY FUNDED BY THE LILLY ENDOWMENT, INC.

ALGEBRA TEST

(SECOND COURSE)

(THIS TEST PREPARED BY THE SECONDARY MATHEMATICS EDUCATION COMMITTEE, DEPARTMENT OF MATHEMATICAL SCIENCES, BALL STATE UNIVERSITY.)

DIRECTIONS FOR TEST:

DO NOT open this booklet until you are told to do so.

This is a test of your competence in the second course of high school algebra. For each of the 50 problems there are listed 5 possible answers. You are to work each problem and determine which is the correct answer, and indicate your choice by filling in the circle in the correct place on the separate answer sheet provided. A sample follows:

	A	B	C	D	E
1.	①	②	③	●	⑤

1. If $x + 2 = 6$, then x equals:

A. 8

B. 3

C. $\frac{1}{3}$

D. 4

E. none of the above

The correct answer for the sample is "4," which is answer D ; therefore, you should answer this question by filling in the circle D as indicated above.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any question. If you are unable to work any particular problem, it is to your advantage to guess at the answer rather than leave it blank. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet and begin work. When you have finished one page, go on to the next page. The working time for the entire test is 90 minutes. Calculators are not allowed.

DIRECTIONS FOR ANSWER SHEET:

Fill in your name in the blanks provided. Above your name write the name of your school and the city where it is located.

Along the side of your name indicate your sex and grade by filling in the circle provided. A sample follows:

SAMPLE:

Mary A. Brown who goes to Western High School in Muncie and is in the eleventh grade would write across the top and fill in along the side.

WESTERN HIGH SCHOOL - MUNCIE, INDIANA

NAME (Last, First, M.I.)

B	R	O	W	N		M	A	R	Y		A			

SEX
Ⓜ
●

GRADE
⑨
⑩
●
⑫

DO NOT TURN PAGE UNTIL YOU ARE TOLD TO DO SO.

1. A line with a y-intercept of 3 and an x-intercept of 5 has the equation

   a. $5x + 3y = 1$

   b. $5x + 3y = 15$

   c. $3x + 5y = 1$

   d. $3x + 5y = 15$

   e. None of the above.

2. What is the slope of the line which is perpendicular to $3x - 7y = 15$?

   a. $\frac{3}{7}$

   b. $-\frac{7}{3}$

   c. $\frac{7}{3}$

   d. $-\frac{3}{7}$

   e. None of the above.

3. Which of the following points is <u>not</u> on the graph of $x^2 - x - 2 > 0$?

   a. $(-2, -2)$

   b. $(-2, -1)$

   c. $(2, 3)$

   d. $(3, 2)$

   e. None of the above.

4. A polynomial $P(x)$ has the property that $P(0) = 0$, $P(1) = 0$, $P(-2) = 0$, and $P(2) = 4$. $P(x)$ is which of the following?

   a. $x^3 + x^2 - 2x$

   b. $x^3 + x^2 - 2x + 4$

   c. $\frac{1}{2}(x^3 + x^2 - 2x - 4)$

   d. $\frac{1}{2}(x^3 + x^2 - 2x)$

   e. None of the above.

5. When $x^4 - 2x^3 + 3x^2 - x + 2$ is divided by $x - 3$ the remainder is

   a. $x^3 + x^2 + 6x + 17$

   b. 167

   c. 53

   d. 0

   e. None of the above.

6. The points $(2, -3)$, $(4, 3)$, and $(5, \frac{k}{2})$ are on the same straight line. The value(s) of k is (are)

   a. 12

   b. -12

   c. $\pm$ 12

   d. 12 or 6

   e. 6 or $6\frac{2}{3}$

7. The polar coordinates of two points are $P_1(\sqrt{2}, \frac{3\pi}{4})$ and $P_2(2, \frac{11\pi}{6})$. The mid-point of the segment joining P_1 and P_2 has what <u>rectangular</u> coordinates?

   a. $(\frac{2 + \sqrt{2}}{2}, \frac{21\pi}{24})$

   b. $(\frac{-1 + \sqrt{2}}{2}, \frac{1}{2})$

   c. $(\frac{1 + \sqrt{2}}{2}, \frac{2 + \sqrt{3}}{2})$

   d. $(\frac{-1 + \sqrt{3}}{2}, 0)$

   e. None of the above.

8. The solutions of the equation $x^{\log_{10} x} = 100x$ are

   a. 0, 100

   b. 1, 100

   c. $\frac{1}{10}$, 100

   d. 10, 100

   e. None of the above.

9. The solution set for the equation $\sqrt{x+1} - \sqrt{x} = 2$ is

   a. $\emptyset$

   b. $\{\frac{9}{4}\}$

   c. $\{\frac{9}{16}\}$

   d. $\{\frac{1}{2}\}$

   e. None of the above.

10. In a given arithmetic sequence the first term is 2, the last term is 29, and the sum of all the terms is 155. The common difference is

   a. 3

   b. 2

   c. $\frac{27}{19}$

   d. $\frac{13}{9}$

   e. $\frac{23}{38}$

11. Which of the following is <u>not</u> a factor of $x^6 - y^6$?

   a. $x - y$

   b. $x + y$

   c. $x^4 + x^2y^2 + y^4$

   d. $x^2 + xy + y^2$

   e. None of the above.

12. Compute: $e^{5 \ln 2}$

   a. 10

   b. 25

   c. $5 \ln 2$

   d. 32

   e. None of the above.

13. If $\log_b 10 = p$ and $\log_b 2 = q$, then $\log_b 20$ is

    a. pq

    b. $p + q$

    c. p^q

    d. q^p

    e. $2(p + q)$

14. A man, when asked his age, replied "If you take one year from my present age the result will be three times my son's age, and three years ago my age was twice what his will be in five years". The man is how old?

    a. 12

    b. 36

    c. 37

    d. 34

    e. None of the above.

15. Assuming that $\frac{a}{b} = \frac{c}{d}$ and $abcd \neq 0$, which of the following is true?

    a. $\frac{a + c}{c} = \frac{b + d}{d}$

    b. $\frac{a}{c} = \frac{b}{d}$

    c. $\frac{a^2 + b^2}{a^2} = \frac{c^2 + d^2}{c^2}$

    d. All of the above.

    e. None of the above.

16. In the arithmetic sequence $-\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, \ldots$, what is the 13th term?

a. $\frac{5}{2}$

b. $\frac{11}{4}$

c. 3

d. $\frac{13}{4}$

e. $\frac{7}{2}$

17. A zero of the function $f(x) = x^3 - 2x^2 - x + 2$ is

a. 1

b. -1

c. 2

d. All of the above.

e. None of the above.

18. If an integer x is divided by another integer y, the quotient is 24. If the sum of the two integers is 75, then x is

a. 3

b. 8

c. 25

d. 48

e. 72

19. The system:
$$\begin{aligned} x + y - z &= 0 \\ 2x - y - 8z &= 0 \\ 3x + 5y + z &= 0 \end{aligned}$$

a. has no solution

b. has just one solution

c. has just three solutions

d. has infinitely many solutions

e. None of the above.

20. If $\log x = \log 1 + \log 2 + \log 3 + \log 4 + \log 5$, then x is

a. 6

b. 15

c. 36

d. 55

e. 120

21. The solutions to $e^{2x} - 3e^{x} + 2 = 0$ are

a. .3010, 0

b. ln 2, 0

c. .3010, $\log_{10} 1$

d. (a) and (c)

e. None of the above.

22. The graph of $x^2 - 4y^2 = 0$

a. is a hyperbola intersecting only the x-axis.

b. is a hyperbola intersecting only the y-axis.

c. is a hyperbola intersecting neither axis.

d. is a pair of straight lines.

e. does not exist.

23. If $m = \sqrt{3}$, then m^{-4} is

    a. -9

    b. $\frac{1}{81}$

    c. $\frac{1}{9}$

    d. 3

    e. 9

24. How many different seating arrangements could be made for six people at a round table?

    a. 120

    b. 360

    c. 720

    d. 4320

    e. None of the above.

25. If $f(x) = 4x + 3$ and $g(x) = x^2 - 2$, then $f(g(x))$ is

    a. $-x^2 + 4x + 5$

    b. $x^2 + 4x + 1$

    c. $4x^2 - 8$

    d. $4x^2 - 5$

    e. $4x^2 + 1$

26. Find the sum of all even integers between 1 and 999.

a. 249,500

b. 499,000

c. 500,000

d. 998,000

e. None of the above.

27. What are all real numbers x for which $(x - 3)(x + 1) > 0$?

a. $x < -1$

b. $x > 3$

c. $-1 < x < 3$

d. $-3 < x < 1$

e. $x < -1, x > 3$

28. The degree of $(x^2 + 1)^4(x^3 + 1)^3$ as a polynomial in x is

a. 5

b. 7

c. 12

d. 17

e. 72

29. The solution set of $y^4 + y^2 - y - 1 = y(y^3 - 1)$ is

a. {1, -1, i, -i}

b. {0, 1, -1}

c. {1, -1}

d. {1}

e. {0}

30. Which of the following could <u>not</u> be a root of the equation

$2x^5 - 4x^4 + 3x^3 + x^2 - x + 7 = 0$?

a. 7

b. -7

c. $-\frac{1}{2}$

d. $\frac{1}{7}$

e. None of the above.

31. The rational zero(s) of $P(x) = 3x^3 + 8x^2 + 19x + 10$ is (are)

a. $\frac{2}{3}, -\frac{2}{3}$

b. $-\frac{1}{3}, \frac{2}{3}$

c. $-\frac{2}{3}$

d. $-\frac{5}{3}$

e. None of the above.

32. Which of the following sequences is a geometric sequence?

a. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

b. 1, 3, 6, 10, 15, ...

c. $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots$

d. $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$

e. $1, \frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \ldots$

33. If $1 - \frac{4}{x} + \frac{4}{x^2} = 0$, then $\frac{2}{x}$ equals

a. -1

b. 1

c. 2

d. -1 or 2

e. -1 or -2

34. A rectangular box without a top is to be made of wood weighing 40 pounds per cubic foot. If the dimensions of the box are to be 2 feet deep, 3 feet wide and 5 feet long, how thick may the wood be if the empty box is not to weigh more than 160 pounds? Answer to the nearest $\frac{1}{4}$ inch.

a. $1\frac{1}{2}$

b. $1\frac{1}{4}$

c. 1

d. $\frac{3}{4}$

e. None of the above.

35. Determine k in $k^2(x^2 + 1) - k(x + 1) = 3x$ so that the product of the roots is six times their sum. K has values

a. -9, 2

b. 9, -2

c. $-\frac{3}{2}, \frac{1}{3}$

d. $\frac{3}{2}, -\frac{1}{3}$

e. None of the above.

36. The sentence $|x - 5| < 4$ is equivalent to which one of the following?

    a. $-4 < x < 4$

    b. $1 < x < 9$

    c. $-9 < x < 9$

    d. $-1 < x < 9$

    e. None of the above.

37. One mason can lay the brick in a certain wall in 45 hours. A second mason requires 30 hours for the same work. How many hours will it take them to do the work if they work together?

    a. 75

    b. 37.5

    c. 18

    d. 15

    e. None of the above.

38. If $2, 6, 18, 3x + 3$ are the first four terms of a geometric sequence, then x equals

    a. 5

    b. 7

    c. 9

    d. 11

    e. 17

39. The expression $\left(\frac{a}{a+y} + \frac{y}{a-y}\right) \Big/ \left(\frac{y}{a+y} - \frac{a}{a-y}\right)$, where a is a non-zero real number has the value -1 for

a. all but two real values of y

b. only two real values of y

c. all real values of y

d. only one real value of y

e. no real values of y

40. If $25^x = 5$ and $3^{(x+y)} = 81$, then y equals

a. 2

b. $\frac{5}{2}$

c. 3

d. $\frac{7}{2}$

e. $\frac{9}{2}$

41. What must the value of k be in the polynomial $9x^2 + 2kx + 16$ if the trinomial is a perfect square?

a. 6

b. 12

c. 24

d. It is impossible for this trinomial to be a perfect square.

e. None of the above.

42. Solve: $\frac{3}{x+2} + \frac{5}{x-3} = \frac{3x+1}{(x+2)(x-3)}$

a. $x = 0$

b. $x = 2$

c. $x = 3$ and $x = -2$

d. no solution

e. None of the above

43. If y varies directly as x, and if $y = 8$ when $x = 4$, the value of y when $x = -8$ is

a. -16

b. -4

c. -2

d. $4k$, $k = \pm 1, \pm 2, \ldots$

e. $16k$, $k = \pm 1, \pm 2, \ldots$

44. If x, $2x + 2$, $3x + 3$, ... are in geometric progression with non-zero common ratio, the fourth term is

a. -27

b. $-13\frac{1}{2}$

c. -12

d. $13\frac{1}{2}$

e. 27

45. If $P(x) = (x + 1)(x^2 + 1)(x^4 + 1)(x^8 + 1)$ then $(x - 1)\,P(x)$ is

a. $x^{16} - x^{15} + x^{14} - x^{13} + x^{12} - x^{11} + x^{10} - x^9 + x^8 - x^7 + x^6 - x^5 + x^4 - x^3 + x^2 - x + 1$

b. $x^{14} - 1$

c. $x^{16} - x^{14} + x^{12} - x^{10} + x^8 - x^6 + x^4 - x^2 + 1$

d. $x^{16} - 1$

e. None of the above.

46. Solve for x in the reals: $(\frac{1}{3})^{2x-1} = 3(9)^{x-1}$

a. 1

b. $-\frac{1}{2}$

c. $\frac{1}{2}$

d. 2

e. None of the above.

47. If $x = 1 + 2^p$ and $y = 1 + 2^{-p}$ then y in terms of x is

a. $\frac{x + 1}{x - 1}$

b. $\frac{x + 2}{x - 1}$

c. $\frac{x}{x - 1}$

d. $2 - x$

e. $\frac{x - 1}{x}$

48. For what value of k is $x - 1$ a factor of $3x^5 - k$?

a. -3

b. $\frac{1}{3}$

c. $\frac{5}{3}$

d. 3

e. 5

49. The eleventh term in the expansion of $(1 - \frac{x}{2})^{32}$ is

a. $\frac{321\ x^{10}}{2^{10}\ 10!\ 22!}$

b. $\frac{32 \cdot 31 \cdot 30 \cdots 23x^{10}}{10!}$

c. $\frac{321\ x^{10}}{2^{10}\ 10!}$

d. $\frac{321\ x^{10}}{10!\ 22!}$

e. None of the above.

50. From the letters of the word ALGEBRA, how many arrangements can be made taking any number of letters from one to seven at a time?

a. 5,040

b. 7,012

c. 21,036

d. 35,280

e. None of the above.

1984

STATE HIGH SCHOOL MATHEMATICS CONTEST

SPONSORED BY THE INDIANA COUNCIL OF TEACHERS OF MATHEMATICS

PARTIALLY FUNDED BY THE LILLY ENDOWMENT, INC.

GEOMETRY TEST

(GEOMETRY TEST PREPARED BY THE STAFF OF THE MATHEMATICS EDUCATION DEVELOPMENT CENTER, INDIANA UNIVERSITY - BLOOMINGTON.)

DIRECTIONS FOR TEST:

DO NOT open this booklet until you are told to do so.

This is a test of your competence in high school geometry. For each of the 30 problems there are listed 5 possible answers. You are to work each problem and determine which is the correct answer, and indicate your choice by filling in the circle in the correct place on the separate answer sheet provided. A sample follows:

1. If $x + 30^{\circ} = 90^{\circ}$, then x equals:

A. 120°

B. 3°

C. -60°

D. 60°

E. none of these

	A	B	C	D	E
1.	①	②	③	●	⑤

The correct answer for the sample is "60°", which is answer D; therefore, you should answer this question by filling in the circle D as indicated above

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any question. If you are unable to work any particular problem, it is to your advantage to guess at the answer rather than leave it blank. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet and begin work. When you have finished one page, go on to the next page. The working time for the entire test is 90 minutes.

DIRECTIONS FOR ANSWER SHEET:

Fill in your name in the blanks provided. Above your name write the name of your school and the city where it is located, including zip code.

Along the side of your name indicate your sex and grade by filling in the circle provided. A sample follows:

SAMPLE: Mary A. Brown who goes to Western High School in Muncie and is in the ninth grade would write across the top and fill in along the side.

WESTERN HIGH SCHOOL - MUNCIE, INDIANA 47306

NAME (Last, First, M.I.)														
B	R	O	W	N		M	A	R	Y		A			

SEX
(M)
●

GRADE
●
(10)
(11)
(12)

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.

1. Given <1 and <3 are complementary and < 1 and <2 are supplementary. Study the following statements:

   I. $m(<2) = m(<3)$.
   II. $m(<2)$ is obtuse.
   III. $m(<2) = m(<3) + 90^{\circ}$.

   Select the correct answer:

   a) I and II are true

   b) II and III are true

   c) only I is true

   d) only III is true

   e) none of the statements are true

2. The sum of the measures of angles A,B,C,D, and E in the accompanying figure is:

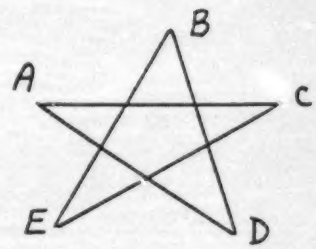
   a) Less than 180°

   b) 180°

   c) Greater than 180° but less than 360°

   d) 360°

   e) Cannot be determined



3. A collapsible, portable ironing board is constructed so that the supports bisect each other. The board will always be parallel to the floor because:

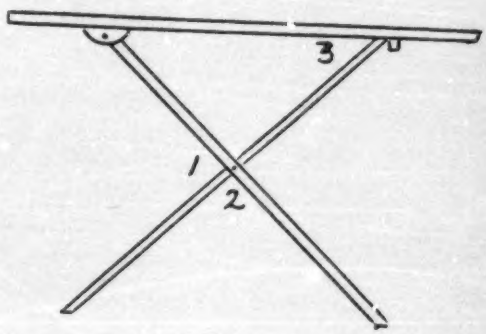
   a) $m(<1) = m(<3)$

   b) If two angles of a triangle are equal, then the sides opposite them are equal.

   c) If the diagonals of a quadrilateral bisect each other, the quadrilateral is a parallelogram.

   d) If two parallel lines are cut by a transversal, the corresponding angles are equal.

   e) $m(<1) + m(<2) = 180^{\circ}$



4. In the diagram below, $\overline{JK} \parallel \overline{TQ}$ and $\overline{RT} \perp \overline{JQ}$. m(<JKO) is:

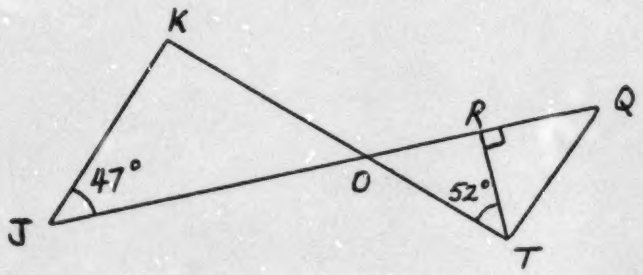
a) 90°

b) 38°

c) 95°

d) 85°

e) 52°



5. In the figure $\overline{AB} \parallel \overline{CD}$. What is the measure of <EFG?

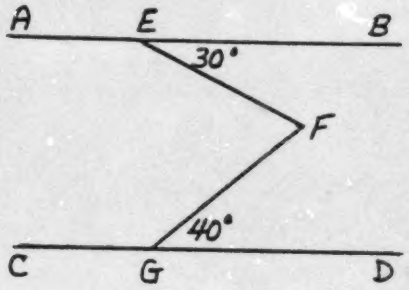
a) 90°

b) 10°

c) 60°

d) 70°

e) cannot be determined.



6. In right triangle ABC, $\overline{DE} \perp \overline{BC}$. If AB = 6, AC = 8, BC = 10 and DE = 4, find EC.

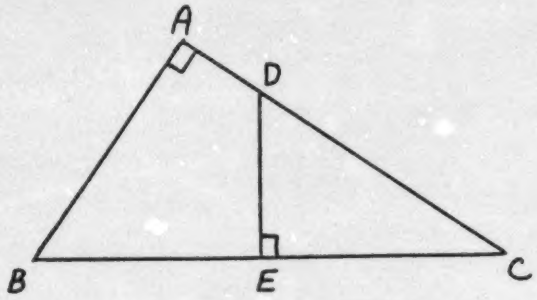
a) $5\frac{1}{3}$

b) $6\frac{2}{3}$

c) 5

d) 6

e) $4\frac{3}{4}$



7. Three cylindrical oil drums of 2-foot diameter are to be securely fastened in the form of a "triangle" by a rope. What length of rope, in feet, will be required?

a) 12

b) 6

c) $6 + 2\pi$

d) $12 + 2\pi$

e) Cannot be determined.

8. How many degrees between the hands of a clock at 3:40?

   a) 125°

   b) 145°

   c) 140°

   d) 135°

   e) 130°

9. Each of the quadrilaterals, A,B,C,D,E,F,G,H and I is a square. If the area of square C is 64 square inches and the area of square D is 81 square inches, the area of square A, in square inches is:

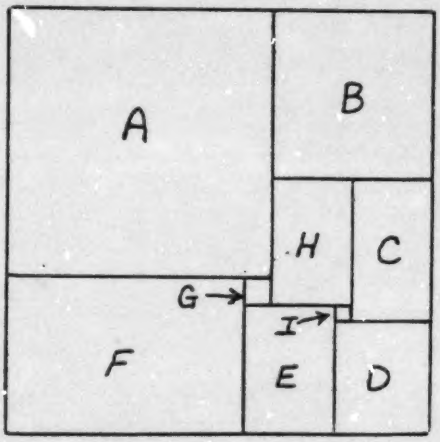
   a) 145

   b) 243

   c) 384

   d) 405

   e) None of the above.



10. An equilateral triangle and a regular hexagon have equal perimeters. If the area of the triangle is 2, then the area of the hexagon is:

    a) 2

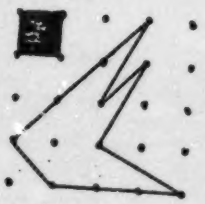
    b) 8

    c) 5

    d) 3

    e) 6

11. If the area of the shaded square is 1 square unit, then the area of the polygon is:



a) $5\frac{1}{2}$ sq. units

b) 7 sq. units

c) $6\frac{1}{2}$ sq. units

d) 5 sq. units

e) $7\frac{1}{2}$ sq. units

12. In a right circular cone, the height measures twice the radius of the base. If the radius is 8 inches, what is the volume of the cone?

a) $\frac{1024\pi}{3}$ in.^{3}

b) 1024π in.^{3}

c) $\frac{128}{3}$ in.^{3}

d) $\frac{2048\pi}{3}$ in.^{3}

e) $\frac{1024}{3}$ in.^{3}

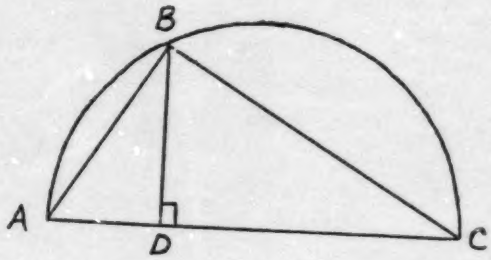
13. Given semi-circle ABC with $\overline{BD}$ perpendicular to the diameter $\overline{AC}$. Study the following conditions:

I. Δ ABC is isosceles.

II. Δ ADB is similar to Δ ABC

III. $\frac{AD}{BD} = \frac{BD}{DC}$

IV. $AC^2 = AB^2 + BD^2 + CB^2$



Select the correct statement:

a) Only I is true.

b) Only I and II are true.

c) Only II and III are true.

d) Only III is true.

e) Only II and IV are true.

14. A triangle which has vertices (0,0), (2,3), and (3,-2) belongs to which of the following sets:

I - Isosceles triangles
II - Right triangles
III - Equilateral triangles

a) none of these

b) I only

c) II only

d) III only

e) I and II only

15. The center of a circle is at the point (1,2) and the point (5,6) is on the circle. The area of the circle is:

a) $8\sqrt{2}\pi$

b) $32\ \pi$

c) $30\ \pi$

d) $14\ \pi$

e) None of the above

16. If the area of a circle is 100 π, what is the length of a side of a regular hexagon which is inscribed in this circle?

a) 20

b) $16\frac{2}{3}$

c) 10

d) $8\frac{1}{3}$

e) None of these

17. Given: "All class officers are members of the student council." Which statement expresses a conclusion that follows logically from this given statement?

a) All members of the student council are class officers.

b) If a student is not a member of the student council, then the student is not a class officer.

c) If a student is not a class officer, he is not a member of the student council.

d) If a student is a member of the student council, he is a class officer.

e) None of these

18. In the figure, $m(<P) = 2(m(<Q))$ and the $m(<QRS) = 108^0$ Triangle PQR is:

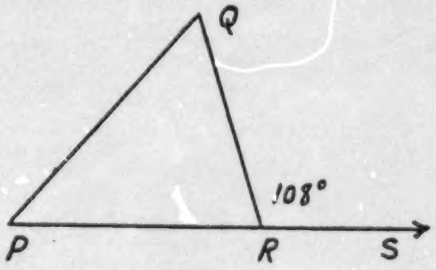
a) isosceles

b) right

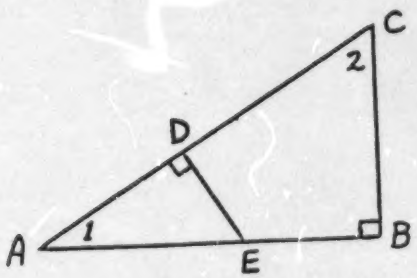
c) obtuse

d) scalene

e) equilateral



19.



In the figure shown, which of the following statements must be true?

a) $\overline{DE} \cong \overline{AD}$

b) $m(<1) = m(<2)$

c) $m(<1) + m(<2) = 180^0$

d) E is the midpoint of $\overline{AB}$

e) none of the above must be true

20. Two different planes, y and z are each perpendicular to plane t. Which of the following statements must be true?

a) Plane y is perpendicular to plane z.

b) The line of intersection of y and t is parallel to the line of intersection of z and t.

c) The line of intersection of y and t is perpendicular to the line of intersection of z and t.

d) If y and z intersect, their line of intersection is perpendicular to t.

e) If y and z intersect, their line of intersection is parallel to t.

21. Quadrilateral ABCD is a rhombus with $\overline{AB}$ of length 5 units and the length of one of its diagonals is 6 units. The area of the rhombus, in square units, is:

a) 25

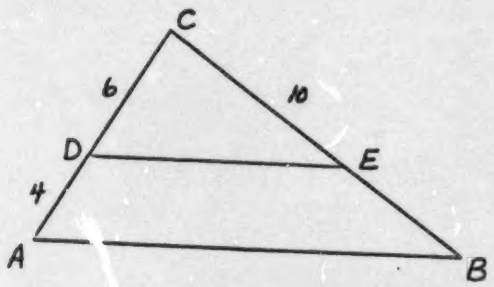
b) 20

c) 12

d) 24

e) 30

22.



In the figure shown, $\overline{DE} \| \overline{AB}$ and segments have the measures shown. The measure of $\overline{BE}$ is:

a) $\frac{50}{3}$

b) 8

c) 18

d) $\frac{20}{3}$

e) 15

23. A boat travels 40 miles east, 80 miles south, then 20 miles east again. How far is it from the starting point?

a) 140

b) 40

c) 50

d) 100

e) 60

24. A pony is tied to the corner of a barn that is 10 meters by 12 meters. If the length of rope is 15 meters, then how much area, in square meters, outside the barn, is available to the pony?

a) $\frac{675\pi}{4}$

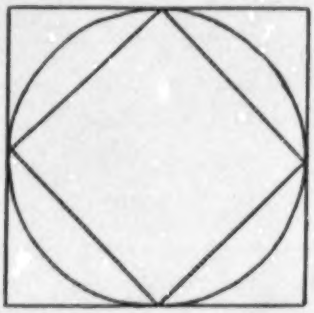
b) $\frac{709\pi}{4}$

c) $\frac{120\pi}{15}$

d) $\frac{225\pi}{4}$

e) None of the above.

25. In the figure, a circle is inscribed in a square and then a smaller square is inscribed in the circle. The ratio of the area of the smaller square to that of the larger square is:



a) 1 : 4

b) $\sqrt{2}$

c) 1 : 2

d) 1 : $\sqrt{2}$

e) 2 : 3

26. A plane is 7 inches from the center of a sphere having a radius of length 13 inches. The area, in square inches of the circle of intersection is:

a) 12π

b) 144π

c) $\sqrt{120\pi}$

d) 120π

e) None of the above.

27. How many cartons 3' x 3' x 3' can be stored in a room 12' x 9' x 8'?

a) 32

b) 30

c) 28

d) 26

e) 24

28. In a triangle ABC, with right angle at C, the perpendicular bisectors of sides $\overline{AC}$ and $\overline{BC}$

a) intersect outside the triangle

b) intersect inside the triangle

c) intersect on the hypotenuse

d) do not intersect

e) cannot be determined

29. If only planes are used to form the surface of a solid, the minimum number needed is:

a) 2

b) 3

c) 4

d) 5

e) 6

30. Each interior angle of a regular polygon exceeds the exterior angle by 150°. The number of sides to the polygon is:

a) 16

b) 18

c) 20

d) 24

e) 25

1984

STATE HIGH SCHOOL MATHEMATICS CONTEST

SPONSORED BY THE INDIANA COUNCIL OF TEACHERS OF MATHEMATICS

PARTIALLY FUNDED BY THE LILLY ENDOWMENT, INC.

COMPREHENSIVE TEST

(COMPREHENSIVE TEST PREPARED BY MEMBERS OF THE MATHEMATICS FACULTY AT IUPU - FORT WAYNE.)

DIRECTIONS FOR TEST:

DO NOT open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each of the 28 problems there are listed 5 possible answers. You are to work each problem and determine which is the correct answer, and indicate your choice by filling in the circle in the correct place on the separate answer sheet provided. A sample follows:

1. If $x + 2 = 6$, then x equals:

A. 8

B. 3

C. $\frac{1}{3}$

D. 4

E. none of these

1. A B C D E
① ② ③ ● ⑤

The correct answer for the sample is "4", which is answer D; therefore, you should answer this question by filling in the circle D as indicated above.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any question. If you are unable to work any particular problem, it is to your advantage to guess at the answer rather than leave it blank. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet and begin work. When you have finished one page, go on to the next page. The working time for the entrie test is 90 minutes.

DIRECTIONS FOR ANSWER SHEET:

Fill in your name in the blanks provided. Above your name write the name of your school and the city where it is located, including zip code.

Along the side of your name indicate your sex and grade by filling in the circle provided. A sample follows:

SAMPLE: Mary A. Brown who goes to Western High School in Muncie and is in the ninth grade would write across the top and fill in along the side.

WESTERN HIGH SCHOOL - MUNCIE, INDIANA 47306

NAME (Last, First, M.I.)															
B	R	O	W	N		M	A	R	Y		A				

SEX
(M)
●

GRADE
●
(10)
(11)
(12)

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.

1. When $x^9 - x$ is factored as completely as possible into polynomials and monomials with real, integral coefficients, the number of factors is:

(A) More than 5.

(B) 5.

(C) 4.

(D) 3.

(E) 2.

2. What is the ratio of the shaded to the unshaded areas?

(A) $\frac{\pi}{1 - \pi}$

(B) $\frac{4 - \pi}{\pi}$

(C) $\frac{\pi}{\pi - 4}$

(D) $\frac{\pi}{4 - \pi}$

(E) $\frac{\pi}{4}$

3. If the reciprical of $x + 1$ is $x - 1$, then x equals:

(A) 0

(B) 1

(C) -1

(D) ± 1

(E) None of the above.

4. Point E is on side AB of square ABCD. If EB has length one, and EC has length two, determine the area of the square.

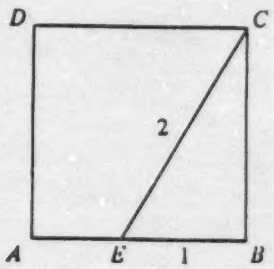
(A) $\sqrt{3}$

(B) 5

(C) 3

(D) $2\sqrt{3}$

(E) $4\sqrt{3}$



5. For what value(s) of k does the pair of equations $y = x^2$ and $y = 3x + k$ have two identical solutions?

(A) $\frac{4}{9}$

(B) $-\frac{4}{9}$

(C) $\frac{9}{4}$

(D) $-\frac{9}{4}$

(E) $\pm\frac{9}{4}$

6. If the parabola $y = ax^2 + bx + c$ passes through the points (-1,12), (0,5), and (2,-3), the value of (a + b + c) is:

(A) -4

(B) -2

(C) 1

(D) 12

(E) 0

7. Consider the graphs of $y = 2\log x$ and $y = \log 2x$. We may say that the equations:

   (A) do not intersect.

   (B) intersect in exactly one point.

   (C) intersect in exactly two points.

   (D) intersect in a finite number of points greater than two.

   (E) coincide.

8. Of the members of three athletic teams in a school, 21 play basketball, 26 play baseball, 29 play football, 14 play basketball and baseball, 15 play baseball and football, 12 play football and basketball, and 8 play all three sports. How many players are there?

   (A) 76

   (B) 125

   (C) 117

   (D) 43

   (E) 58

9. The graph of $x^2 - 4y^2 = 0$ is:

   (A) a parabola.

   (B) an ellipse.

   (C) a pair of straight lines.

   (D) a point.

   (E) none of the above.

10. What is the smallest positive integer by which 540 should be multiplied so that the product is a perfect cube?

(A) $(540)^2$

(B) 90

(C) 120

(D) 50

(E) 5

11. How many terminating zeroes are in the number 25! (25 factorial)?

(A) 5

(B) 6

(C) 3

(D) 2

(E) 8

12. Evaluate: $\sin(2\arccos(\frac{20}{29}))$.

(A) $\frac{840}{841}$

(B) $\frac{21}{29}$

(C) $\frac{41}{841}$

(D) $\frac{20}{21}$

(E) $\frac{9}{29}$

13. Find the area of the region included between the graphs of the following:

$$x^2 + y^2 = 4 \quad \text{and} \quad |x| + |y| = 2$$

(A) $\pi - 4$

(B) $4 - \pi$

(C) $4\pi - 8\sqrt{2}$

(D) 8

(E) $4\pi - 8$

14. If the graphs of $2y + x + 3 = 0$ and $3y + ax + 2 = 0$ are to meet at right angles, the value of a is:

(A) $\pm \frac{2}{3}$

(B) $-\frac{2}{3}$

(C) $-\frac{3}{2}$

(D) 6

(E) -6

15. Let the set consisting of the squares of the positive integers be called S. If a certain operation on one or more members of a set always yields a member of the set, we say that the set is <u>closed</u> under that operation. Therefore S is closed under:

(A) addition.

(B) multiplication.

(C) division.

(D) extraction of a positive integral root.

(E) none of the above.

16. The sides of a triangle are 4cm, 5cm, and 7cm. Find the area of the triangle in $(cm)^2$.

(A) 10

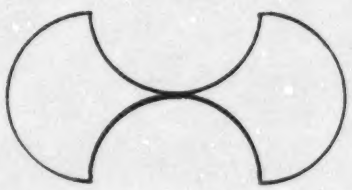
(B) 17.5

(C) 14

(D) $4\sqrt{6}$

(E) $5\sqrt{6}$

17. The given figure is constructed of four semicircles, two of which are tangent at their midpoints. If the diameter of each circle is one unit, find the area in square units of the figure.



(A) π

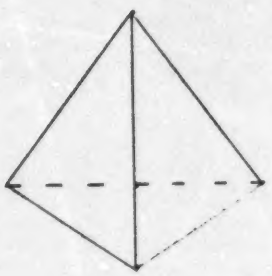
(B) $\pi\sqrt{2}$

(C) $\frac{\pi}{4}$

(D) 1

(E) Not enough information.

18. Find the length of the major altitude of a regular tetrahedron whose side is six units long.



(A) $6\sqrt{2}$

(B) $\frac{1}{2}\sqrt{117}$

(C) $2\sqrt{6}$

(D) $2\sqrt{3}$

(E) 8

19. If $\log_6 7 = a$, and $\log_6 5 = c$, express $\log_{25} 7$ in terms of a and c.

(A) $\frac{2c}{a}$

(B) $\frac{a}{2c}$

(C) $\frac{2a}{c}$

(D) $\frac{c}{2a}$

(E) $2ac$

20. If $a + b = 1$, and $a^2 + b^2 = 2$, determine the value of $a^3 + b^3$.

(A) $\frac{5}{2}$

(B) $\frac{7}{2}$

(C) 2

(D) 3

(E) $\frac{3}{2}$

21. If $\underline{n}$ men working $\underline{n}$ hours a day for $\underline{n}$ days produce $\underline{n}$ articles, how many articles will be produced by $\underline{x}$ men working $\underline{y}$ hours a day for $\underline{w}$ days?

(A) $\frac{xyw}{n^2}$

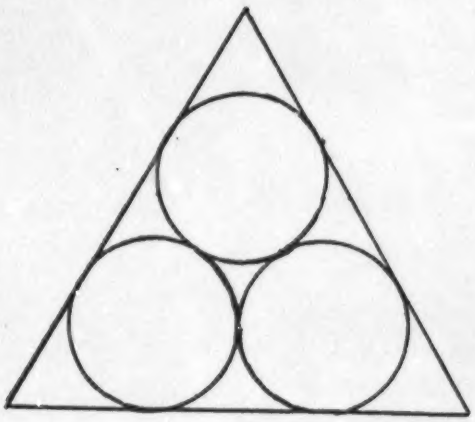
(B) $\frac{xy}{w}$

(C) $\frac{xyw}{n^3}$

(D) $\frac{wy}{x}$

(E) $\frac{xyw}{n}$

22. In the given figure, each of the three circles is tangent to the other two, and each side of the triangle is tangent to two of the circles. If each circle has radius 3, determine the perimeter of the triangle.



(A) $36 + 9\sqrt{3}$

(B) 45

(C) $36 + 6\sqrt{3}$

(D) $18 + 18\sqrt{3}$

(E) None of the above.

23. The first three terms of an arithmetic progression are $x - 1$, $x + 1$, $2x + 3$ in the order shown. The value of x is:

(A) -2

(B) 0

(C) 2

(D) 4

(E) undetermined

24. Let n be the number of pairs (x,y) which satisfy $5y - 3x = 15$ and $x^2 + y^2 \leq 16$, with x and y real numbers. Then n is:

(A) 0

(B) 1

(C) 2

(D) greater than two, but finite.

(E) an infinite number.

25. Determine the value of x in the given figure.

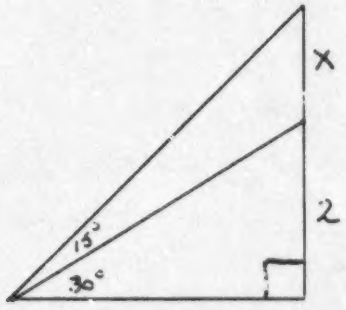
(A) $2\left(\frac{\sqrt{3}}{3} - 1\right)$

(B) 1

(C) $\sqrt{3}$

(D) $\sqrt{3} - 1$

(E) $2(\sqrt{3} - 1)$



26. A square sheet of tin is formed into an open box by cutting from each corner a square of side S inches and then turning up the sides. If the box is to contain c cubic inches, find the length of a side of the original sheet of tin.

(A) $\sqrt{c} - S$

(B) $\sqrt[3]{c - S}$

(C) $\sqrt[3]{cS}$

(D) $\sqrt{\frac{c}{S}} + 2S$

(E) Not enough information.

27. Find all possible real numbers y so that for some real number x, $(x \neq 1)$, $x^y = 1$.

(A) 0

(B) 1

(C) all odd integers

(D) all even integers

(E) all integers

28. Determine the sum of all the positive, three digit integers formed from the digits 2, 3, 5, 6, 7, if no digit is repeated when forming each number. (For example, 235 and 532 are two of the numbers, but 233 is not.)

(A) 27,600

(B) 30,636

(C) 32,760

(D) 276

(E) 30,000